Equations

Previous Year Questions:

1. In the set of Integers, the equation $3x + 6y = 11$ has ______.
   (a) Infinitely many solutions
   (b) No solutions
   (c) Unique Solution
   (d) Data Inadequate

   Solution:
   $3x + 6y = 11$
   $3(x + 2y) = 11$
   $x + 2y = \frac{11}{3}$
   According to question both x & y are integers.
   For integers, it is true that:
   Integer + Integer=Integer,
   But in the question,
   Integer + Integer = Fraction,
   Which is not possible.
   Hence, no integral solution exists.

2. The number of values of k for which the system of equations:
   $(k + 1)x + 8y = 4k$
   $kx + (k + 3) = 3k - 1$
   has infinitely many solutions, is:
   (a) 0 (b) 1
   (c) 2 (d) Infinite

   Solution:
   $$\begin{array}{ccccc} (k+1)x & + & 8y & = & 4k \\ kx & + & (k+3)y & = & 3k-1 \end{array}$$
   For no solution,
   $\frac{k+1}{k} = \frac{8}{k+3}$
   = $(k + 1)(k + 3) = 8k$
   = $k^2 - 4k + 3 = 0$
   = $k = 3$ or $k = 1$
   For, $k = 3$,
   The equation becomes,
   $$\begin{array}{ccccc} 4x & + & 8y & = & 12 \\ 3x & + & 6y & = & 8 \end{array}$$
   $$\begin{array}{ccccc} x & + & 2y & = & 3 \\ x & + & 2y & = & \frac{8}{3} \end{array}$$
   ⇨ No solution
   For, $k = 1$,
   ⇨ $$\begin{array}{cccc} 2x & 8y & = & 4 \\ x & 4y & = & 2 \end{array}$$
   ⇨ $$\begin{array}{ccccc} x & + & 4y & = & 2 \\ x & + & 4y & = & 2 \end{array}$$
   ⇨ Infinite solutions
   So, there is only one value of 'k', for which no solution exists for the given equation.
   Hence, option (b) is the correct answer.

3. Kunal told Kanika that he has either a or b marbles. Further he stated that a, b satisfy the equation $x^2 - p(x + 1) + c = 0$, then $(a + 1)(b + 1) =$
   (a) $1 - c$ (b) $c - 1$
   (c) $1 + c$ (d) c

   Solution:
   ∵ a & b satisfy the equation
   $x^2 - p(x + 1) + c = 0$
   = $x^2 - px + (-p + c) = 0$
   ⇨ $(a + b) = p$
   $ab = (-p + c)$
   To find $(a + 1)(b + 1)$
   = $ab + a + b + 1$
   = $-p + c + p + 1$
   = $c + 1$
   Hence, option (c) is the correct answer

4. The positive value of m for which the roots of equation $x^2 + 4(m - 2)x + 27 = 0$ are in ratio 1:3, is:
   (a) 7 (b) 5
   (c) 3 (d) 1

   Solution:
   $x^2 + 4(m - 2)x + 27 = 0$
   Here, product of roots $= 27$
   Sum of roots $= -4(m - 2)$
   ∵ roots are in the ratio $1:3$

$\therefore x$ & $3x$ can be taken as the roots

⇨ Product of roots = 27

= $3x^2 = 27$

= $x = \pm 3$

For, $x = 3$, second root = 9

⇨ Sum of roots = 12

$4(m-2) = 12$

$m = 5$

Or,

For, $x = -3$, second root = − 9

Sum of roots = -12

$4(m-2) = -12$

= $m = -1$

$\because m = +ve$,

$\therefore m = 5$

Hence, option (b) is the correct answer

Alternatively,

Directly use option (b).

$x^2 + 4(m-2)x + 27 = 0$

$x^2 + 4(5-2)x + 27 = 0$

$(x+3)(x+9) = 0$

$x = -3 \; or \; x = -9$

⇨ Ratio of roots = 1: 3 as given in the question

Hence, option (b) is the correct answer

5. The lines $y = (a+1)x + 3 \; and \; y = -3x + 2$ are parallel if a =

(a) −4 (b) 4
(c) −3 (d) 3

Solution:

$y = (a+1)x + 3$

$y = -3x + 2$

If the two lines are parallel,

Then, $(a+1) = -3$

$a = -4$

Hence, option (a) is the correct answer

6. The system of linear equations:

(4d −1)$x + y + z = 0$

$-y + z = 0$and (4d −1) $z = 0$

has a trivial solution, if d equals:

(a) 1/2 (b) 1/4
(c) 3/4 (d) 1

Solution:

$$\begin{matrix} (4\mathrm{d}-1)\mathrm{x} & + & \mathrm{y} & + & \mathrm{z} & = & 0 \\ & - & \mathrm{y} & + & \mathrm{z} & = & 0 \\ & & & & (4\mathrm{d}-1)\mathrm{z} & = & 0 \end{matrix}$$

For trivia solutions,

$$\begin{vmatrix} (4d-1) & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & (4d-1) \end{vmatrix} = 0$$

Expand along C_1 and solve.

$d = \frac{1}{4}$

Hence option (b) is the correct answer

7. The value of a so that -2 is a root of $2x^2 - x + a = 0$ is:

(a) 10 (b) −10
(c) 9 (d)−9

Solution:

Equation = $2x^2 - x + a = 0$

Root$_1$ = -2

⇨ $x = -2$

Solving the equation at $x = -2$

We get $a = -10$

Hence, option (b) is the correct answer

8. The solution to the equation $2(4)^{x+1} = 4(8)^{x+2}$ is:

(a) −5 (b)1
(c)0 (d) No solution

Solution:

= $2(4)^{x+1} = 4(8)^{x+2}$

= $2(2^2)^{x+1} = 2^2(2)^{3(x+2)}$

= $2^{2x+2+1} = 2^{3x+6+2}$

$\Rightarrow 2x + 3 = 3x + 8$

= $x = -5$

Hence option (a) is the correct answer

9. If $a^2 - b^2 = 18$, what is the value of (a + b) if (a – b) = 9

(a) 2 (b) 9
(c) 27 (d) None of these

Solution:
= $a^2 - b^2 = 18$
= $(a-b)(a+b) = 18$
= $(a-b) = 9$
$\Rightarrow (a+b) = 2$
Hence, option (a) is the correct answer

10. $(1+\sqrt{3})(1-\sqrt{3})$ is equal to:-
(a) – 2 (b) 2
(c) – 3 (d) – 1

Solution:
$(1+\sqrt{3})(1-\sqrt{3})$
= $1 - 3$
= -2
Hence, option (a) is the correct answer

11. If $x \propto yz$ and $y \propto xz$ then,
(a) $z \propto ay$ (b) $z \propto \frac{y}{x}$
(c)z is constant (d) $z \propto \frac{x}{y}$

Solution:
$x \propto yz \;\rightarrow x = kyz$ ..... (1)
$y \propto xz \;\rightarrow y = kxz$ ...... (2)
Put value of x from (1) in (2)
$y = kz.kzy$
= $1 = k^2z^2$
= $\frac{1}{k^2} = z^2$
$\because k = constant,$
$\therefore z = constant$
Hence option (c) is the correct answer

Logarithm

Previous Year Questions:

1. If $a^2 + b^2 = 7ab$ then
(a) alog $(a+b) = \log a + \log b$
(b) alog $\frac{(a+b)}{2} = \frac{1}{3}(\log a + \log b)$
(c) $\log\frac{(a+b)}{3} = \frac{1}{2}(\log a + \log b)$
(d) $\log(3(a+b)) = 2(\log a + \log b)$

Solution:
= $a^2 + b^2 = 7ab$
= $a^2 + b^2 + 2ab = 7ab + 2ab$
= $a^2 + b^2 + 2ab = 9ab$
= $(a+b)^2 = 9ab$
= $(a+b) = \sqrt{9ab}$
= $(a+b) = 3\sqrt{ab}$
Use option (c)
= $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a + \log b)$
= $\log\left(\frac{3\sqrt{ab}}{3}\right) = (\log a + \log b)$
= $\log\sqrt{ab} = \log a + \log b$
= $\frac{1}{2}\log(a+b) = \log a + \log b$
Hence option (c) is correct

2. The value of $2^{\log_3 5} - 5^{\log_3 2}$ is:
(a) 0 (b) 1
(c) 2 (d) 3

Solution:
Value of $2^{\log_3 5} - 5^{\log_3 2}$
Let, $\log_3 5 = a$
= $3^a = 5$
Let, $\log_3 2 = b$
= $3^b = 2$
⇨ $3^{blog_3 5} - 3^{alog_3 2}$
= $3^{\log_3 5^b} - 3^{\log_3 2^a}$
= $5^b - 2^a$
i.e.
= $5^{\log_3 2} - 2^{\log_3 5}$
Which means,
$2^{log_3 5} - 5^{log_3 2} = 5^{log_3 2} - 2^{log_3 5}$
⇨ $2^{\log_3 5} = 5^{\log_3 2}$
So, $2^{\log_3 5} - 5^{\log_3 2} = 0$
Hence, option (a) is the correct answer.

3. If a, b, c are three successive terms of a geometric series of positive real numbers and $x > 0, \log_a x, \log_b x, \log_c x$ are in
(a) AP (b) GP
(c) HP (d) None of these

Solution:

Let, $a = 2, b = 4, c = 8, d = 64$
Then, $\log_2 64 = 6$
= $\log_4 64 = 3$
= $\log_8 64 = 2$
Since, 6, 3, 2 are neither in AP nor in GP
So, they may be in HP
Check for HP
$\because \frac{1}{6}, \frac{1}{3}, \frac{1}{2}$ must be in AP
Then , 6, 3 and 2 are in HP
Hence the answer is option (c)

4. If $\log_8 \log_{13}(\sqrt{x+13} + \sqrt{x}) = 0$, what is the value of x?
(a) 16 (b) 36
(c) 23 (d) 26

Solution:
= $\log_8\left(\log_3\left(\sqrt{x+13} + \sqrt{x}\right)\right) = 0$
= $\left(\log_3\left(\sqrt{x+13} + \sqrt{x}\right)\right) = 1$
= $\left(\sqrt{x+13} + \sqrt{x}\right) = 13$
Using option (b),
= $\sqrt{49} + \sqrt{36}$
= $7 + 6$
=13
Hence option (b) is the correct answer.

5. If $\log_{0.1} x^2 > \log_{0.1} 25$ then x is:
(a) $(-\infty, -5)$ (b) $(-5, 5)$
(c) $(-\infty, -5) \cup (5, \infty)$ (d) $(5, \infty)$
Solution:
If $\log_{0.1} x^2 > \log_{0.1} 25$
Since, the base of the logarithmic terms is less than 1,
Therefore, if higher the power of 0.1 will be, lesser would be the number formed.
⇨ $x^2 < 25$
The above condition is true only for option (b)

6. If $\log_4 64 + \log_3 9 = \log_{10} x$, then x is equal to
(a) 10^5 (b) 10^7
(c) 10^{12} (d) 10^{15}

Solution:
= $\log_4 64 + \log_3 9 = \log_{10} x$
= $3 + 2 = \log_{10} x$
= $\log_{10} x = 5$
= $x = 10^5 = 100000$
Hence, option (a) is the correct answer

7. If $\log_b(5) = c$, then b^{4c} will be:
(a) 4 (b) 5
(c) 25 (d) 625

Solution:
= $\log_b 5 = c$
= $b^c = 5$
= $b^{4c} = 5^4$
= 625
Hence, option (d) is the correct answer

8. The domain of $f(x) = In(2-x) + ln(3-x)$ is given by:
(a) $(-\infty, 2)$ (b) (2,3)
(c) $(-\infty, 3)$ (d) $(-\infty, 2]$

Solution:
= $f(x) = \ln(2-x) + \ln(3-x)$
Since the term in the logarithmic function cannot be less than or equal to zero,
= $(2-x) > 0 \,\&(3-x) > 0$
$\Rightarrow x < 2$
So, the domain is $(-\infty, -2)$
Hence, option (a) is the correct answer

9. If $\log_2 \log_4(\sqrt{x+4} + \sqrt{x)} = 0$, then what is the value of ?
(a) 1/4 (b) 3/4
(c) 9/4 (d) Cannot be solved

Solution:
= $\log_2 \log_4\left(\sqrt{x+4} + \sqrt{x}\right) = 0$
= $\log_4(\sqrt{x+4} + \sqrt{x}) = 1$
= $\sqrt{x+4} + \sqrt{x} = 4$
Use option (c) we get LHS = RHS
Hence, option (c) is the correct answer.

10. Solve for x when $\frac{\log_3 9^x + \log_{81} 9}{\log_9 81} = \frac{1}{4}$

(A) 0 (B) 1
(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Solution:

$= \frac{\log_3 9^x + \log_{81} 9}{\log_9 81} = \frac{1}{4}$

$= \frac{\log_3 3^{2x} + \log_{81} 81^{\frac{1}{2}}}{\log_9 9^2} = \frac{1}{4}$

$= \frac{2x+\frac{1}{2}}{2} = 1/4$

$= x = 0$

Hence, option (a) is the correct option

Functions and Sets

Previous Year Questions:

1. Let $\mathcal{R}$ be the relation over the set of natural numbers such that $m\mathcal{R}n \leftrightarrow m$ divides n. Then $\mathcal{R}$ is
(a) Reflexive and Symmetric
(b) Symmetric and transitive
(c) Reflexive and Transitive
(c) Equivalence Relation

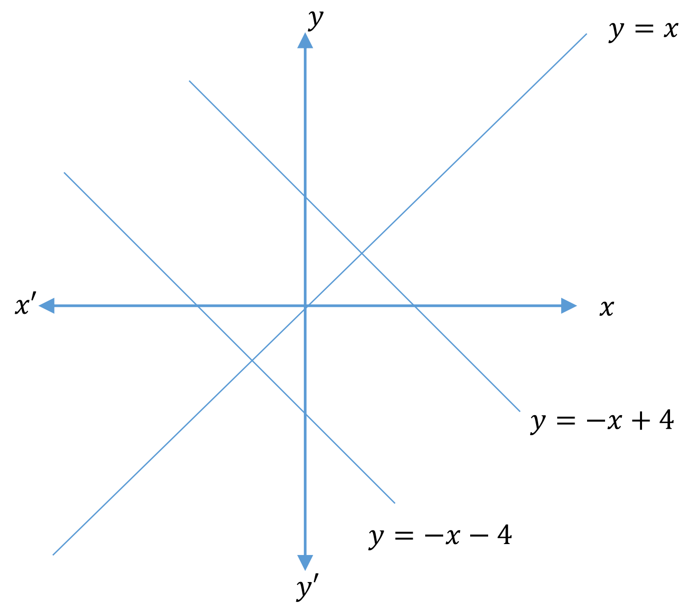
Solution:
$mRn \leftrightarrow m\ divides\ n$
It is read as
Let R be the relation over sets m and n such that m divides n
Let, m and n be two sets each with three numbers a, b & c, where a divides b & b divides c.
Now,
∵ a divides a, b divides b & c divides c
∴ The relation R will contain the elements $(a,a),(b,b),(c,c)$
Hence the relation is reflexive
∵ a divides b and b divides c,
∴ a divides c
i.e. If (a,b) and (b,c) are present in the relation then (a,c) also exists
Hence the relation is transitive
But, if a divides b, it does not mean that b divides a.
i.e. If (a,b) exists, (b,a) will not exist(till $a = b$)
Hence the relation is not symmetric
Hence option (c) is the correct answer

2. If the point (a,a) falls between the lines $|x+y| = 4$ then
(a) $|a| < 2$ (b) $|a| = 2$
(c) $|a| < 1$ (d) $|a| < \frac{1}{2}$

Solution:
$|x+y| = 4$

$|x+y| = 4 = \begin{Bmatrix} -(x+y) & = & 4 \\ (x+y) & = & 4 \end{Bmatrix}$

The graph for the above equations will be:



The point (a,a) will lie in the line $y = x$ as both x & y coordinates are equal.
⇨ The portion of the line $x = y$ lying between $|x+y| = 4$ bounded by the line $x = 2$ and $x = -2$
∴ the line $|x| = 2$ is the boundary
⇨ $|a| = 2$ is the required answer
Hence option (b) is the correct answer

3. Which of the following is true?
(a) The set $\{x: x+8 = 8\}$ is a null set
(b) $\{1\} \subseteq \{\{1,2\},\{2\},3\}$
(c) $2 \in \{\{1,2\},\{2\},3\}$
(d) None of the above

Solution:
Option (c) is the correct answer

4. Let $A = \{\phi, \$\}, B = \{a, b, l, a\}$ then the number of distinct relations from A to B is
(a) 8 (b) 9
(c) 2 (d) 6

Solution:
$A = \{\phi, \$\}$
$B = \{a, b, l\}$ (Since, an element cannot be written twice in a set)
So, number of relations from A to B
= $n(A) \times n(B)$
= $2 \times 3 = 6$
Hence option (d) is the correct answer

5. In an exam, the average was found to be 50 marks. After deducting computational errors the marks of 100 candidates got to be changed from 90 to 60 each & the average came down to 45 marks. The total number of candidates who took the exam were:
(a) 200 (b) 300
(c) 600 (d) 150

6. Rohan found that for a function f, $f(x) = 3x - 5$ and $f\left(g(x)\right) = 2x$, then the function $g(x) =$
(a) $2x$ (b) $x + 3$
(c) $\frac{2x+5}{3}$ (d) 2

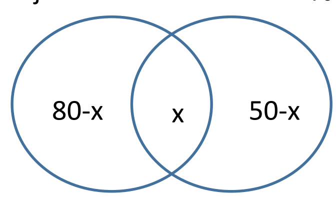
Solution:
= $f(x) = 3x - 5$
= $f\left(g(x)\right) = 2x$
Use option (c).
⇨ Let $g(x) = \frac{2x+5}{3}$
So, $f\left(g(x)\right) = 3\left(\frac{2x+5}{3}\right) - 5 = 2x$
Hence, option (c) is the correct answer

7. The value of the function $f(x) = ax^2 + bx + 2$ at 1 is 3 and at 4 is 42, then b is:
(a) 4 (b) 3
(c) -1 (d) -2

Solution:
Given $f(x) = ax^2 + bx + 2$
= $f(1) = 3$
= $f(4) = 42$
i.e. $a(1)^2 + b(1) + 2 = 3$
& $a(4)^2 + b(4) + 2 = 42$
on solving
= $a = 3$
= $b = -2$
Hence, option (d) is the correct answer

8. In a class test consisting of Maths and Physics, 80% passed in Maths and 50% passed in Physics 15% failed in both subjects. If 180 students passed in both the subjects how many students failed in both the subjects?
(a) 80 (b) 60
(c) 90 (d) 50

Solution:
Percentage of students passed in atleast one subject = $100 - 15 = 85\%$



- $85 = 80 + 50 - \mathrm{x}$
- $\mathrm{X} = 45\%$

9. In a locality, two thirds of the people have cable TV, one-fifth have VCR, and one-tenth have both. The fraction of people having either cable-TV or VCR is:-
(a) 19/30 (b) 3/5
(c) 17/30 (d) 23/30

Solution:
For the given question, let us assume the number of people to be a multiple of 3,5 & 10.

Let the number of people be = 300
People having TV $= \frac{2}{3} \times 300 = 200$
People having VCR $= \frac{1}{5} \times 300 = 60$
People having both $= \frac{1}{10} \times 300 = 30$
People having only TV $= 200 - 30 = 170$
People having only VCR $= 60 - 30 = 30$
People having both $= 30$
Total number p=of people having either TV or VCR $= 170 + 30 + 30 = 230$
Fraction of people having either TV or VCR $= \frac{230}{300}$
Hence, option (d) is the correct answer

10. If two sets A and B are defined as $A = \left\{(x, y)|y = \frac{1}{x}, 0 \neq \in \mathbb{R}\right\}$ and $B = \{(x, y)|y = -x,\ x \in \mathbb{R}\}$, then
(a) $A \cap B = A$
(b) $A \cap B = B$
(c) $A \cap B = \phi$
(d) None of these

Solution:
= $A = \left\{(x, y) \,\middle|\, y = \frac{1}{x}, 0 \neq \epsilon\, \mathcal{R}\right\}$
= $B = \{(x, y) \mid y = -x, x\, \epsilon\, \mathcal{R}\}$
= $A =$
$\left\{\ldots\left(-3, -\frac{1}{3}\right), \left(-2, -\frac{1}{2}\right), (-1, -1), (1,1), \left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right) \ldots\right\}$
= $B =$
$\{\ldots(-3,3), (-2,2), (-1,1), (0,0), (1, -1), (2, -2), (3, -3) \ldots\}$
= $A \cap B = \phi$
Hence, option (c) is the correct answer.

11. What is the possible number of reflexive relations on a set of 4 elements?
(a) 2^4
(b) 2^8
(c) 2^{12}
(d) 2^{16}

Solution:
The total number of reflexive relations on a set of n elements is : $2^{n^2 - n}$
So, for 4 elements, it is $2^{4^2-4} = 2^{12}$
Hence (c) is the correct answer

12. The range of the function $f(x) = \frac{x}{1+x^2}$, for $x \in R$, is:
(a) $[-1/2, 0) \cup (0, 1/2]$
(b) $(-1/2, 0) \cup (0, 1/2]$
(c) $[-1/2, 1/2]$
(d) $[0, 1/2]$

13. For whole number x, if $f(x) = f(x-1) + f(x-2), f(0) = 1. f(1) = 1$, then $(f \circ f)(5)$ is equal to:
(a) 34
(b) 55
(c) 21
(d) 8

Solution:
= $f(x) = f(x-1) + f(x-2)$
= $f(0) = 1, f(1) = 1,$
Finding $f(2)$
= $f(2) = f(1) + f(0)$
= $f(2) = 1 + 1 = 2$
Similarly, $f(3) = f(2) + f(1) = 2 + 1 = 3$
= $f(4) = f(3) + f(2) = 3 + 2 = 5$
= $f(5) = f(4) + f(3) = 5 + 3 = 8$
Hence, option (d) is the correct answer

14. Let f: $[-1, 1] \to [-1,1]$ be defined by $f(x) = \sin \pi x$. Then the function f is:
(a) Neither one-one nor onto
(b) Onto but not one-one
(c) One-one but not onto
(d) Both one-one and onto

15. Let f, g: $R - R$ defined by $f(x) = 2x + 1$ and $g(x) = x/3$. Then $(f^{-1} \circ g^{-1})(x)$ is equal to:
(a) $\frac{3x - 1}{2}$
(b) $\frac{3x\,(x - 1)}{2}$
(c) $\frac{x - 1}{2}$
(d) $\frac{x - 3}{6}$

Solution:
= $f(x) = 2x + 1, \quad g(x) = \frac{x}{3}$
= $f^{-1}\ o\ g^{-1}(x)$
Finding $f^{-1}(x)$
= $f(x) = 2x + 1$
=> $x = \frac{f(x)-1}{2}$
So, $f^{-1}(x) = \frac{x-1}{2}$

Similarly,
= $g(x) = \frac{x}{3}$
=> $x = 3(g(x))$
So, $g^{-1}(x) = 3x$
= $f^{-1}(g^{-1}(x)) = \frac{g^{-1}(x)-1}{2}$
= $\frac{3x-1}{2}$
Hence, option (a) is the correct answer

16. If $f(x) = \frac{x-1}{x+1}$, then $f(2x)$ is:
(a) $\frac{2f(x)+1}{f(x)+3}$ (b) $\frac{f(x)+1}{f(x)-1}$
(c) $\frac{3f(x)+1}{f(x)+1}$ (d) None of these

Solution:
= $f(x) = \frac{x-1}{x+1}$
= $f(2x) = \frac{2x-1}{2x+1}$
Using option(a)
= $\frac{2f(x)+1}{2f(x)+3}$ = $\frac{2\left(\frac{x-1}{x+1}\right)+1}{\left(\frac{x-1}{x+1}\right)+3}$ = $\frac{3x-1}{4x+2}$
Using option (b) and solving as above, we get:
= $\frac{f(x)+1}{f(x)-1}$ = $-x$
Using option (c) we get:
= $\frac{2x-1}{x}$
Hence, option (d) is the correct answer

17. At the point of inflection on the function $y = f(x)$
(a) the value of $f(x)$ changes from positive to negative or vice versa
(b) the slope of $f(x)$ changes from positive to negative or vice versa
(c) the curvature of $f(x)$ changes from positive to negative or vice versa
(d) All of the above

Solution:
At the point of inflection on the function $y = f(x)$ the slope of $f(x)$ changes from positive to negative or vice versa.
Hence option (b) is correct.

18. The minimum value of $|x^2 - 5x + 21|$ is:
(A) -5 (B) 0
(C) -1 (D) -2

Solution:
Minimum value of $|x^2 - 5x + 21|$
∵ The given function is a modulus function, its value never goes $-ve$.
Hence, $(a), (c) \& (d)$ can never be the answers.
Hence, option (b) is the correct answer

19. A car is accelerating uniformly and its velocity is given as a linear function of time t. The velocity of the car $V(t)$ and the time t are given in the table below

t	1	3	4	9
$V(t)$	5	a	b	21

The values of a and b are:
(a) 9 & 11
(b) 11 & 15
(c) 11 & 14
(d) 9 & 16

Solution:
Since, velocity of a vehicle is dependent on time,
∴ $v(t) = xt + y$
At $t = 1, v(t) = 5$
At $t = 9, v(t) = 21$
i.e. $5 = x(1) + y$
= $21 = x(9) + y$
⇨ $x = 2 \& y = 3$
∴ $v(t) = 2t + 3$
At $t = 3,\ v(3) = 9$
At $t = 4,\ v(4) = 11$
Hence, option (a) is the correct answer

20. X and Y are two variables. The corresponding values of X and Y are given below:

X	3	6	9	12	24
Y	24	12	8	6	3

Then the relationship between X and Y is given by (where $'\propto'$ stands for proportionality)

(a) $X+Y \propto X-Y$

(b) $X+Y \propto \frac{1}{X-Y}$

(c) $X \propto Y$

(d) $X \propto \frac{1}{Y}$

Solution:

It can be clearly seen that as x rises, the value of y decreases and as y increases, the value of x decreases.

So, $x \propto \frac{1}{y}$

Hence, option (d) is the correct answer

Limits and Continuity

Past Year Questions

1. $\lim\limits_{x\to 0} \frac{3^x-2^x}{4^x-9^x}$ is :

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) 1 (d) -1

Solution:

$= \lim\limits_{x\to 0} \frac{3^x-2^x}{4^x-9^x}$

$= \lim\limits_{x\to 0} \frac{3^x-2^x}{-(9^x-4^x)}$

$= \lim\limits_{x\to 0} \frac{3^x-2^x}{-(3^x-2^x)(3^x+2^x)}$

$= \lim\limits_{x\to 0} \frac{1}{-(3^x+2^x)}$

$= -\frac{1}{2}$

Hence, option (b) is the correct answer

2. What value of 'b' should be assigned to make the function

$$g(x) = \begin{cases} x^3, & x < \frac{1}{2} \\ bx^2, & x \geq \frac{1}{2} \end{cases}$$

continuous at $x = \frac{1}{2}$

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) 1 (d) 0

Solution:

$= g(x) = \begin{cases} x^3 & x < \frac{1}{2} \\ bx^2 & x \geq \frac{1}{2} \end{cases}$

Finding LHL,

$= \lim\limits_{x\to \frac{1}{2}^-} x^3 = \lim\limits_{h\to 0} \left(\frac{1}{2}-h\right)^3 = \frac{1}{8}$

Finding RHL,

$= \lim\limits_{x\to \frac{1}{2}^+} bx^2 = \lim\limits_{h\to 0} b\left(\frac{1}{2}+h\right)^2 = \frac{b}{4}$

For continuity, LHL=RHL

⇨ $\frac{1}{8} = \frac{b}{4}$

$= b = \frac{1}{2}$

Hence, option (b) is the correct answer

3. The points of discontinuity for $\boldsymbol{f(x)} = \left(\frac{\mathbf{1}}{\mathbf{\log|x|}}\right)$ are

(a) $0. \pm 1$ (b) $1, -1$

(c) $0, 1$ (d) $0, -1$

Solution:

$= f(x) = \frac{1}{\log|x|}$

The point of discontinuity will be the point(s) where:

(a) Denominator = 0

Or,

(b) Denominator does not exist

We know, that the $\log(x)$ function does not exist at $x = 0$

And that the value of $\log|x| = 0$ at $x = \pm 1$

So, the points of discontinuity are:

$(0, +1, -1)$

i.e. $(0, \pm 1)$

Hence, option (a) is the correct answer

4. The function $f(x) = |x|, -1 \leq x \leq 2$ is:

(a) Neither continuous nor differentiable at origin

(b) Continuous but not differentiable at origin

(c) Continuous and differentiable at all points on the given interval

(d) None of these

Solution:
The function $f(x) = |x|, -1 \leq x \leq 2$ is continuous but not differentiable at origin

5. Consider the following two statements about the function $f(x) = |x|$
P: $f(x)$ is continuous for all real values of x
Q: $f(x)$ is differentiable for all real values of x
(a) P is true and Q is false
(b) P is false and Q is true
(c) Both P and Q are true
(d) Both P and Q are false

Solution:
f(x) is continuous everywhere but not differentiable at $x = 0$.

6. The value of $\underset{x \to 0+}{Lim} \frac{e^{1/x} - 1}{e^{1/x} + 1}, x \neq 0$, is:
(a) 0 (b) – 1
(c) 1 (d) None of these

Solution:
Let $\frac{1}{x} = y$
If $x \to 0$, then $y \to \infty$

$$\therefore \lim_{x\to 0} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} = \lim_{h\to\infty} \frac{e^h-1}{e^h+1} = \lim_{h\to\infty} \left(\frac{e^h\left(1-\frac{1}{e^h}\right)}{e^h\left(1+\frac{1}{e^h}\right)}\right)$$

$$= \lim_{h\to\infty} \left(\frac{\left(1-\frac{1}{e^h}\right)}{\left(1+\frac{1}{e^h}\right)}\right) = \frac{1-0}{1+0} = 1 \quad \because \lim_{h\to\infty} \frac{1}{e^h} = 0$$

Hence, option (c) is correct

7. $\lim_{x\to -2} \frac{\sqrt{2x+5}-1}{x+2}$
(a) 1 (b) 0
(c) ∞ (d) $-\infty$

Solution:

$$= \lim_{x\to -2} \frac{\sqrt{2x-5}-1}{x+2}$$

$$= \lim_{x\to 2} \frac{\sqrt{2x-5}-1}{x+2} \times \frac{\sqrt{2x-5}+1}{\sqrt{2x-5}+1}$$

$$= \lim_{x\to 2} \frac{2x-5+1}{(x+2)(\sqrt{2x-5}+1)}$$

$$= \lim_{x\to 2} \frac{2(x+2)}{(x+2)(\sqrt{2x-5}+1)}$$

$$= \lim_{x\to 2} \frac{2}{\sqrt{2x-5}+1}$$

= 1
Hence, option (c) is the correct answer

8. Find the value of $\boldsymbol{k}$ so that

$$f(x) = \begin{cases} \frac{x^2-16}{x-4} & x\neq 4 \\ k & x=4 \end{cases}$$

is continuous for all $\boldsymbol{x}$, will be:
(a) Any value (b) 0
(c) 8 (d) 16

Solution:

$$= f(x) = \left\{\begin{matrix} \left(\frac{x^2-16}{x-4}\right) & x \neq 4 \\ k & x = 4 \end{matrix}\right\}$$

Finding LHL

$$\lim_{x\to 4^-} \frac{x^2-16}{x-4} = \lim_{x\to 4^-} (x+4) = \lim_{h\to 0} (4-h+4)$$

= 8
Finding RHL

$$\lim_{x\to 4^+} \frac{x^2-16}{x-4} = \lim_{x\to 4^+} (x+4) = \lim_{h\to 0} (4+h+4)$$

= 8
For continuity,

$$[LHL]_{x=4^-} = [RHL]_{x=4^+} = f(4)$$

⇨ $f(4) = 8$
⇨ $k = 8$

Differentiation

Previous Years Questions:

1. If f be a continuous function on the closed interval [0,2]. If $2 \leq f(x) \leq 4$. Thenthe greatest possible value of $\int_0^2 f(x)\,dx =$
(A) 0 (B) 2
(C) 4 (D) 8

Solution:

$$\int_0^2 f(x) = f(x) \times (2-0) = 4 \times 2 = 8$$

2. If $y = \ln\left(\frac{e^x}{e^x-10}\right)$, then $\frac{dy}{dx} =$
(A) $x - \frac{e^x}{e^x-10}$ (B) $-\frac{1}{e^x}$
(C) $\frac{10}{10-e^x}$ (D) 0

Solution:

Differentiating both sides w.r.t x

$\frac{dy}{dx} = \frac{d}{dx}\left(\ln\left(\frac{e^e}{e^x-10}\right)\right)$

$= \frac{1}{\frac{e^x}{e^x-10}} \times \frac{d}{dx}\left(\frac{e^x}{e^x-10}\right)$

$= \frac{e^x-10}{e^x} \times \frac{(e^x-10)(e^x)'-(e^x)(e^x-10)'}{(e^x-10)^2}$

$= \frac{e^x-10}{e^x} \times \frac{((e^x-10)(e^x)-(e^x)(e^x))}{(e^x-10)^2}$

$= \frac{10}{10-e^x}$

Hence answer is option (c)

3. The value of $\int_1^4 |x-3|\, dx$ is

(a) $-3/2$ (b) $3/2$

(c) $9/2$ (d) $5/2$

Solution:

To find the value of $\int_1^4 |x-3| dx$

First, break the modulus function into two parts (by finding the break point)

⇨ $x - 3 = 0$

⇨ $x = 3$ (this is the break point of the given modulus function)

If, $x < 3$

⇨ $x - 3 = -ve$

If $x = 3$,

⇨ $x - 3 = 0$

If, $x > 3$

⇨ $x - 3 = +ve$

The new function is

$= \int_1^3 -(x-3)dx + \int_3^4 (x-3)\, dx$

$= \left[-\left(\frac{x^2}{2} - 3x\right)\right]_1^3 + \left[\left(\frac{x^2}{2} - 3x\right)\right]_3^4$

$= \left[\left(3x - \frac{x^2}{2}\right)\right]_1^3 + \left[\left(\frac{x^2}{2} - 3x\right)\right]_3^4$

$= \frac{5}{2}$

Hence answer is option (d)

4. The area of the region enclosed by $y = x^2$ and $2y = x$ is:

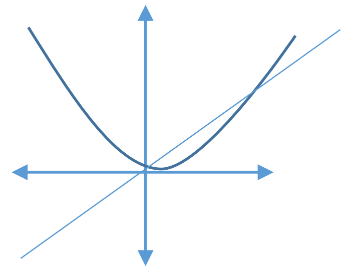
(a) $1/48$ (b) $1/24$

(c) $1/16$ (d) $1/12$

Solution:

The area of the region enclosed by the curves $y = x^2$ & $2y = x$:

First, let us find the points of the given graph:



Function: $y = x^2$

x	0	1	-1	2	-2
y	0	1	1	4	4

Function: $2y = x$

x	0	2	4	6	8
y	0	1	2	3	4

Finding the points of intersection:

$y = [x^2]_{(a,b)}$ (Where, $b = 2a \because y = \frac{x}{2}$)

⇨ $y = [x^2]_{\left(x,\frac{x}{2}\right)}$

⇨ $\frac{x}{2} = x^2$

⇨ $x\left(x - \frac{1}{2}\right) = 0$

⇨ $x = 0$ or $x = \frac{1}{2}$

So, the points of intersection are at $x = 0$ and $x = \frac{1}{2}$

Area between the curves is:

$= [Area]_{line} - [Area]_{curve}$

$= \int_0^{\frac{1}{2}} \frac{x}{2}\, dx - \int_0^{\frac{1}{2}} x^2 dx$

$= \left[\frac{x^2}{4} - \frac{x^3}{3}\right]_0^{\frac{1}{2}}$

$= \frac{1}{16} - \frac{1}{24}$

$= \frac{1}{48}$

Hence, answer is option (a)

5. The function $f(x) = x^3 - 3x$ is:

(a) Increasing on $(-\infty, -1) \cup (1, \infty)$ and decreasing on $(-1, 1)$

(b) Decreasing on $(-\infty, -1) \cup (1, \infty)$ and increasing on $(-1, 1)$

(c) Increasing on $(0, \infty)$ and decreasing on $(-1, 1)$

(d) Decreasing on $(0,\infty)$ and increasing on $(-1,1)$

Solution:
$f(x) = x^3 - 3x$
$f'(x) = 3x^2 - 3$
$= 3(x-1)(x+1)$
⇨ Critical points = $x = \pm 1$

DOMAIN	$f'(x)$	Nature of $f'(x)$
$(-\infty,-1)$	$+ve$	Increasing
$(-1,+1)$	$-ve$	Decreasing
$(+1,+\infty)$	$+ve$	Increasing

⇨ The function is increasing in the domain $(-\infty,-1) \cup (+1,+\infty)$
Hence, option (a) is correct.

6. If $y = \frac{1}{1+x^{n-m}+x^{p-m}} + \frac{1}{1+x^{m-n}+x^{p-n}} + \frac{1}{1+x^{m-p}+x^{n-p}}$ then $\frac{dy}{dx}$ is:
(a) x^{n-p-1} (b) 0
(c) x^{n-m-1} (d) x^{m-p-1}

Solution:
$y = \frac{1}{1+x^{n-m}+x^{p-m}} + \frac{1}{1+x^{m-n}+x^{p-n}} + \frac{1}{1+x^{m-p}+x^{n-p}}$
$y = \frac{x^m}{x^m+x^n+x^p} + \frac{x^n}{x^m+x^n+x^p} + \frac{x^p}{x^m+x^n+x^p}$
$y = 1$
⇨ $\frac{dy}{dx} = 0$
Hence option (b) is correct

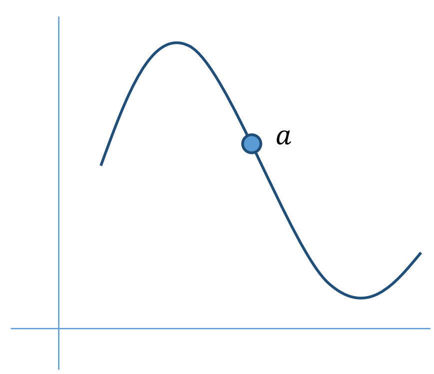
7. The profit of a company is given by $P(x) = 100 + 24x - 18x^2$. The maximum profit that the company can make is:
(a) 76 (b) 108
(c) 80 (d) 110

Solution:
Profit $P(x) = 100 + 24x - 18x^2$.
$P'(x) = 24 - 36x$
For maximum profit, $P'(x) = 0$
⇨ $x = \frac{2}{3}$
Profit at $x = \frac{2}{3}$
$P\left(\frac{2}{3}\right) = 100 + 24\left(\frac{2}{3}\right) - 18\left(\frac{2}{3}\right)^2$
⇨ Maximum profit = 108
Hence option (b) is correct

8. The value of $\int_{-1}^{2} \frac{|x|}{x} dx$ is:
(a) 1 (b) -1
(c) 0 (d) 2

Solution:
$\int_{-1}^{2} \frac{|x|}{x} dx$
Breaking the modulus function
$$|x| = \begin{Bmatrix} -1 & x < 0 \\ 0 & x = 0 \\ +1 & > 0 \end{Bmatrix}$$
$$⇨ \frac{|x|}{x} = \begin{Bmatrix} -\frac{x}{x} & x < 0 \\ \frac{0}{x} & x = 0 \\ +\frac{x}{x} & x > 0 \end{Bmatrix}$$
$$⇨ \frac{|x|}{x} = \begin{Bmatrix} -1 & x < 0 \\ 0 & x = 0 \\ +1 & x > 0 \end{Bmatrix}$$
⇨ $\int_{-1}^{2} \frac{|x|}{x} dx = \int_{-1}^{0} -1 dx + \int_{0}^{2} \frac{|x|}{x} dx$
$= -1 + 2$
$= 1$
Hence answer is option (a)

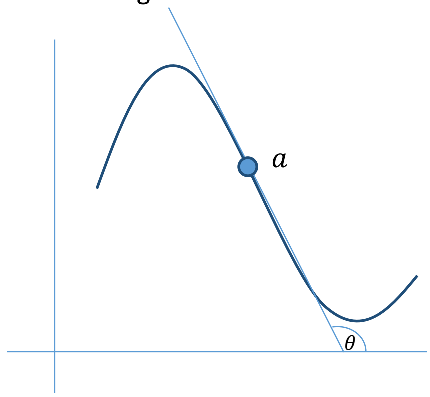
9. For the graph of f(x) depicted, what is the value of $\frac{df(x)}{dx}$ at the point 'a'?



(a) zero (b) positive
(c) Negative (d) None of these

Solution:

According to definition, $\frac{d}{dx}f(x) =$ Slope of curve or tangent to the curve



So, the tangent to the curve at point 'a' will be as shown in the diagram above.

And, it can be clearly seen that $\theta > 90°$

⇨ $\tan\theta = -ve$ (slope is negative)

So, $\left[\frac{d}{dx}f(x)\right]_{x=a} = -ve$

Hence option (c) is correct.

Alternatively,

Another reason for the slope to be $-ve$ is that as we go from $-x$ to $+x$, the value of the function is decreasing. Hence the function is decreasing at point 'a'.

Hence option (c) is correct.

10. The minimum value of $4^x + 4^{1-x}$, for all x$\in R$, is:

(a) 1 (b) 2

(c) 3 (d) 4

Solution:

$y = 4^x + 4^{1-x}$

$\frac{dy}{dx} = 4^x \log 4 - 4^{1-x}\log 4$

$\frac{dy}{dx} = 0$

$(4^x - 4^{1-x})\log 4 = 0$

$4^x = 4^{1-x}$

$x = 1 - x$

$x = \frac{1}{2}$

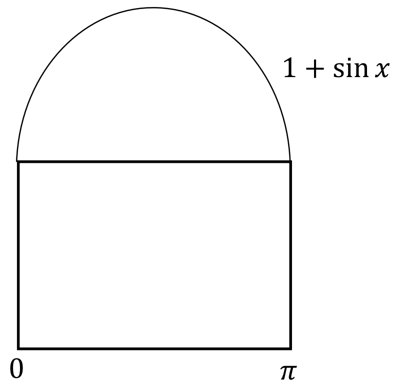
So, the value of function at $x = \frac{1}{2}$

$y = 4^{\frac{1}{2}} + 4^{1-\frac{1}{2}}$

$y = 4$

Hence answer is option (d)

11. A window is in the shape as shown below:



If the cost of glass is Rs.20 per sq unit, then the cost of covering the window with the glass is:

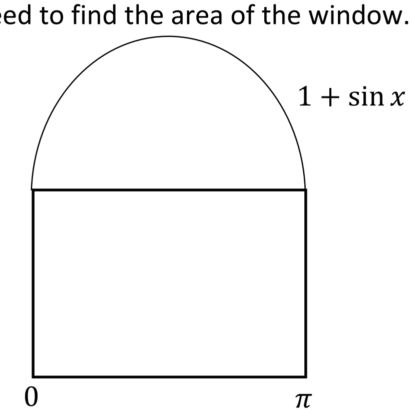
(a) Rs. $(20\pi + 40)$ (b) Rs. $(20\pi + 20)$

(c) Rs. $(\pi + 20)$ (d) Rs. $(2\pi + 40)$

Solution:

Rate of glass = Rs. 20 per sq unit

To fins the total cost of the window, we need to find the area of the window.



Area of window = $\int_0^{\pi}(1 + \sin x)\,dx$

= $\int_0^{\pi} 1dx + \int_0^{\pi} \sin x \; dx$

=$\int_0^{\pi} dx + 2\int_0^{\frac{\pi}{2}} \sin x\, dx$

= $\pi + 1$

= $2 + \pi$

So, cost of glass = $20 \times (2 + \pi)$

= $20\pi + 40$

Hence, option (a) is correct

12. The real number x when added to its inverse gives the minimum value of the sum at x equal to

(a) 1 (b) 2

(c) -2 (d) -1

Solution:

$y = x + \frac{1}{x}$

$\frac{dy}{dx} = 1 - \frac{1}{x^2}$

For critical points, $\frac{dy}{dx} = 0$

$1 - \frac{1}{x^2} = 0$

⇨ $x = \pm 1$

To check for maxima or minima,

$\frac{d^2y}{dx^2} = \frac{2}{x^3}$

$\left[\frac{d^2y}{dx^2}\right]_{x=1} = +ve$

$\left[\frac{d^2y}{dx^2}\right]_{x=-1} = -ve$

So, we get a relative minima at $x = 1$ and relative maxima at $x = -1$

Hence option (a) is the correct answer.

Permutation and Combination

Previous Years Questions:

1. The sum $^nC_1 + 2.^nC_2 + 3.^nC_3 + ... + n.^nC_n$ equals:

(a) $n2^n$ (b) $(n-1)2^{n-1}$

(c) $n2^{n-1}$ (d) $(n-1)2^n$

Solution:

$^nC_1 + 2\,^nC_2 + 3\,^nC_3 + 4\,^nC_4 + \cdots + n\,^nC_n$

Take $n = 1$

We get $^nC_1 = \,^1C_1 = 1$

Look at the options

Option (a) at n=1 = 2

Option (b) at n=1 = 0

Option (c) at n=1 = 0

Option (d) at n=1 = 1

Hence, option (c) is the correct answer.

2. The coefficients of $x^2$and x^3 in the expansion of $[3 + ax]^9$ are equal, then the value of a is

(a) 6

(b) $\frac{1}{3}$

(c) $\frac{3}{7}$

(d) $\frac{9}{7}$

Solution:

= $(3 + ax)^9$

Finding the general term of the series

= $T_{r+1} = \,^9C_r 3^{9-r}(ax)^r$

= $T_{r+1} = \,^9C_r 3^{9-r}a^r x^r$

Here the coefficient term is $^9C_r 3^{9-r}a^r$

For $x^3, r = 3$

Coefficient term = $^9C_3 3^6 a^3$

For $x^2, r = 2$

Coefficient term = $^9C_2 3^7 a^2$

According to question:

$^9C_3 3^6 a^3 = \,^9C_2 3^7 a^2$

On solving,

= $\frac{9!}{7!2!}3^7 = \frac{9!}{6!3!}3^6 a = a = \frac{9}{7}$

Hence, option (d) is the correct answer

3. The sum of $^nC_0 + ^nC_1 + ^nC_2 + ... + ^nC_2$ is

(a) n^n

(b) $n!$

(c) 2^n

(d) $2n!$

Solution:

= $^nC_0 + \,^nC_1 + \,^nC_2 + \cdots + \,^nC_n$

Let $n = 3$

We get,

= $^3C_0 + \,^3C_1 + \,^3C_2 + \,^nC_3$

= $1 + 3 + 3 + 1$

= 8

Check the options.

Option (a) = 27

Option (b) = 6

Option (c) = 8 (Matches the value obtained)

Option (d) = 12

Hence, option (c) is the correct answer

4. If $^nC_{r-1} = 36$, $^nC_r = 84$ and $^nC_{r+1} = 126$, then r is equal to:
(a) 1
(b) 2
(c) 3
(d) None of these

Solution:
$^nC_{r-1} = 36$ ….. (1)
$^nC_r = 84$ ……….(2)
$^nC_{r+1} = 126$ …   (3)
Divide (1) by (2)
= $\frac{r}{n-r+1} = \frac{36}{84}$
= $84r = 36n - 36r + 36$
= $120r - 36n = 36$ …… (A)
Divide (2) by (3)
= $\frac{r+1}{n-r} = \frac{84}{126}$
= $126r + 126 = 84n - 84r$
= $84n - 210r = 126$ …… (B)
Solving (A) and (B)
$n = 9, r = 3$
Hence, option (c) is the correct answer

5. How many triangles can be formed by joining 12 points, of which 7 are collinear?
(a) 220   (b) 35
(c) 185   (d) 84

Solution:
Total number of triangles forms by 12 points = $^{12}C_3$ (as only 3 points are needed to make a triangle)
Total number of triangles forms by 7 points = $^7C_3$
Total number of triangles formed = $^{12}C_3 - {}^7C_3 = 185$
Hence, option (c) is the correct answer

6. The coefficient of x^4 in the expansion of $\left(\frac{x}{2} - \frac{3}{x}\right)^{10}$ is:
(a) $405/64$ (b) $-405/16$
(c) $-18/16$ (d) $81/64$

Solution:
Given expression = $\left(\frac{x}{2} - \frac{3}{x}\right)^{10}$
= $T_{r+1} = {}^{10}C_r \left(\frac{x}{2}\right)^{10-r} \left(-\frac{3}{x}\right)^r =$
$^{10}C_r \frac{(-3)^r}{2^{10-r}} x^{10-2r}$
Here coefficient part = $^{10}C_r \frac{(-3)^r}{2^{10-r}}$ & Variable part = x^{10-2r}
For coefficient of x^4
= $x^{10-2r} = x^4$
= $r = 3$
So, coefficient = $^{10}C_3 \frac{(-3)^3}{2^{10-3}} = -\frac{405}{16}$
Hence, option (b) is the correct answer

7. How many words can be formed with the letters of the word 'EQUATION'?
(a) 6 !   (b) 7 !
(c) 8 !   (d) 9 !

Solution:
Word given = EQUATION
Total number of alphabets = 8
Total number of ways in which new words can be formed using 8 alphabets = 8!

8. The number of words that can be made from the word IMPORTANT in which both the Ts are not together is
(a) 9!-8!   (b) $\frac{9!}{2!} - 8!$
(b) (c) $\frac{9!}{2!} - \frac{8!}{2!}$ (d) $9! - \frac{8!}{2!}$

Solution:
Word given = IMPORTANT
Total number of alphabets = 9 (including 2 Ts)
Total number of ways in which new words can be formed using 9 alphabets out of which 2 are same = $\frac{9!}{2!}$
If both Ts are put together and are considered as a single unit,
I M P R A N [T T]
So, total number of we alphabets = 8
Total number of words that can now be formed = 8!

Total number of ways in which the two Ts can be arranged = $\frac{2!}{2!}$

So, Total number of ways in which both the Ts are not together

$= \frac{9!}{2!} - \left(8! \times \frac{2!}{2!}\right)$

$= \frac{9!}{2!} - 8!$

Hence, option (b) is the correct answer

9. In a group of 6 male executives and 4 female executives, four executives are to be selected for making a team. In how many different ways, can they be selected such that at least one male executive should be there?

(a) 159 (b) 194
(c) 205 (d) 209

Solution:

Males = 6

Females = 4

∵ there should be at least one male executive in 4 members,

Total possible ways are:

Males	Females	Number of ways
1	3	${}^6C_1 \times {}^4C_3$
2	2	${}^6C_2 \times {}^4C_2$
3	1	${}^6C_3 \times {}^4C_1$
4	0	${}^6C_4 \times {}^4C_0$

Total number of ways = $({}^6C_1 \times {}^4C_3) + ({}^6C_2 \times {}^4C_2) + ({}^6C_3 \times {}^4C_1) + ({}^6C_4 \times {}^4C_0)$

Hence, option (d) is the correct answer.

Probability

Previous Years Questions:

1 There are 3 Mathematics, 4 Physics and 1 Chemistry books and they are placed in a shelf. The probability that the books of the same subject is placed together is:

(a) 3/140 (b) 1/40
(c) 9/70 (d) 1/18

Solution:

Mathematics = 3

Physics = 4

Chemistry = 1

Total = 8

Total number of ways in which the books can be arranged = 8!

If all Mathematics, Physics and Chemistry books are kept together,

Number of ways in which the 3 sets of Mathematics, Physics and Chemistry books can be arranged = 3!

Number of ways in which Mathematics books can be arranged = 3!

Number of ways in which Physics books can be arranged = 4!

Number of ways in which Chemistry books can be arranged = 1!

Total number of ways in which books of same subject can be arranged = $3! \times 3! \times 4! \times 1!$

P(Same subject books are together) = $\frac{3!\times3!\times4!\times1!}{8!} = \frac{3}{140}$

Hence, option (a) is the correct answer

2. There are 6 tickets to theatre, four of which are for seats in the front row. 3 tickets are selected at random. What is the probability that two of them are for the front row?

(a) 0.6 (b) 0.7
(c) 0.9 (d) 1/3

Solution:

Front row tickets = 4

Back row tickets = 2

Total tickets = 6

Number of ways of selecting 3 tickets out of 6 tickets = ${}^6C_3$

Number of ways of selecting 2 tickets of front row out of 4 = ${}^4C_2$

Number of ways of selecting 1 ticket of back row out of 2 = ${}^2C_1$

P(2tickets are of front row) = $\frac{{}^4C_2\times\, {}^2C_1}{{}^6C_3} = \frac{6}{10} = 0.6$

Hence, option (a) is the correct answer

3. If a fair die is rolled twice, then the conditional probability that the number 2 has appeared at least once, given that the sum of the numbers is 7, is:

(a) $1/5$ (b) $1/6$

(c) $1/4$ (d) $1/3$

Solution:

Possible ways in which two dice can show a sum of 7 is:

(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) = 6 ways

Out of these possible ways there are only two possible ways in which the digit 2 appears.

Probability = $\frac{2}{6} = \frac{1}{3}$

Hence, option (d) is the correct answer

4. A random variable X, taking values 0, 1, 2 has the following probability distribution

$$P(X) = \begin{cases} K & if\ X = 0 \\ 2K & if\ X = 1 \\ 3K & if\ X = 2 \end{cases}$$

For some number K. What is the value of K?

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) 1 (d) $\frac{1}{5}$

Solution:

X	0	1	2
$P(X)$	k	$2k$	$3k$

∵ Sum of probabilities = 1

⇨ $k + 2k + 3k = 1$

= $k = \frac{1}{6}$

Hence, option (a) is the correct answer

5. A 3 digit number is to be formed using digits 1, 2, 3 & 4 with repetition. What is the probability that it is divisible by 4?

(a) $\frac{1}{4}$ (b) $\frac{1}{4^2}$

(c) $\frac{1}{64}$ (d) $\frac{1}{8}$

Solution:

For being divisible by 4, last two digit number of the number must be divisible by 4.

∴ Last 2 digits must be 12, 24, 32, 44

Since we have 4 digits available, so, the hundredth digit place can be occupied by 4 digits.

Total number of 4 digit numbers= 4×4 = 4^2

Total possible digits = 4^3

P(number divisible by 4) = $\frac{4^2}{4^3} = \frac{1}{4}$

Hence option (a) is correct.

6. A committee of 5 students is to be formed from 6 girls and 4 boys. The probability that the committee has exactly 2 boys is:

(a) $\frac{5}{12}$

(b) $\frac{1}{7}$

(c) $\frac{10}{21}$

(d) $\frac{11}{24}$

Solution:

Boys = 4

Girls = 6

Total = 10

Number of committee members to be selected = 5

Possible ways of selecting 5 members out of 10 = ${}^{10}C_5$

If 2 boys are selected, then 3 girls will also be selected.

Total number of ways of doing the same = ${}^4C_2 \times {}^6C_3$

P(exactly 2 boys) = $\frac{{}^4C_2\times\, {}^6C_3}{{}^{10}C_5} = \frac{10}{21}$

Hence, option (c) is the correct answer

7. The probability that a boy gets scholarship is 0.9 and that a girl will get it is 0.8. The probability that at least one of them gets scholarship is:

(a) 0.75

(b) 0.50
(c) 0.98
(d) 0.90

Solution:
$P(B) = 0.9$
$P(G) = 0.8$
⇨ $P(\bar{B}) = 1 - 0.9 = 0.1$
⇨ $P(\bar{G}) = 1 - 0.8 = 0.2$
∵ Getting scholarships is an independent event,
∴ Probability that both do not get scholarship = $P(\bar{B}) \cap P(\bar{G}) = P(\bar{B}).P(\bar{G}) = 0.1 \times 0.2 = 0.02$
P(at least one of them gets scholarship)= $1 - 0.02 = 0.98$
Hence, option (c) is the correct answer

8. For two events A and B, $P(A) = 0.6$, $P(B) = 0.3, P(A \cup B) = 0.9$. Events A and B are
(a) Independent
(b) Mutually exclusive
(c) Exhaustive
(d) Independent and Exhaustive

Solution:
= $P(A) = 0.6$
= $P(B) = 0.3$
= $P(A \cup B) = 0.9$
∵ $P(A \cup B) = P(A) + P(B)$
∴ A & B are mutually exclusive events

9. If it is known that two heads appeared on two tosses of a fair coin, what is the probability of heads on the 4th toss?
(a) $\frac{2}{3}$ (b) $\frac{1}{4}$
(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Solution:
Since, the appearance of a head or tail on a coin is an independent event, and it does not depend on the previous values appeared,
Therefore, P(Head on 4th toss) = P(Head) = $\frac{1}{2}$

Hence, option (c) is the correct answer

Sequences and Series

Previous Year Questions

1. The value of $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-\sqrt{4}}$ is:
(a) 5 (b) Greater than 5
(c) Less than 5 (d) 0

Solution:
= $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-\sqrt{4}}$
= $\frac{3+\sqrt{8}}{3-8} - \frac{(\sqrt{8}+\sqrt{7})}{8-7} + \frac{(\sqrt{7}+\sqrt{6})}{7-6} - \frac{(\sqrt{6}+\sqrt{5})}{6-5} + \frac{(\sqrt{5}+\sqrt{4})}{5-4}$
= $\frac{3+\sqrt{8}}{-5} + (2 - \sqrt{8})$
= $\frac{13-4\sqrt{8}}{5}$
= $2.6 - \frac{4\sqrt{8}}{5}$
= $\approx 1.6 \left(\because \frac{4\sqrt{8}}{5} \approx 1\right)$
It can be noted that the answer is < 5
Hence option (c) is the correct option.

2. If x, y, z are three natural numbers in arithmetic progression and $x + y + z = 21$, then the possible number of values of the ordered triplet (x, y, z) is:
(a) 13 (b) 14
(c) 15 (d) 12

Solution:
= $S = 1^2 - 2^2 + 3^2 - 4^2 + \cdots + 199^2 - 200^2$
= $1 - 4 + 9 - 16 + 25 - 36 + \cdots$
= $-3 - 7 - 11 - 15 - \cdots - 399$
= $S_n = \frac{n}{2}[a + l]$
= $\frac{100}{2}[-3 - 399]$
= -20100
Hence, option (d) is the correct answer

3. The sum of the series $S = 1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots\ldots + 199^2 - 200^2$, is
(a) 20,000 (b) 20, 100

(c) – 20, 000 (d) – 20, 100

Solution:
$a+b=20$
$\sqrt{ab}=0.8\left(\frac{a+b}{2}\right)$
$\sqrt{ab}=0.4(20)$
$\sqrt{ab}=8$
$ab=64$
$a(20-a)=64$
$a=4, \rightarrow b=16$
or $a=16 \rightarrow b=4$
so, $|a-b|=12$
Hence option (b) is correct

1. The sum of two positive numbers is 20. If the geometric mean of these numbers is 20% less than their arithmetic mean, then the difference between the number is:
(a) 16 (b) 12
(c) 8 (d) 4

Solution:
Let the two numbers be x and y.
$\therefore x+y=20$
AM= $\frac{x+y}{2}=10$, GM = $\sqrt{xy}$
Acc/Que
$\sqrt{xy}=10-20\%\text{ of }10$
$\Rightarrow (x-y)^2=(x+y)^2-4xy$
$=20^2-4\times 64$
$=144$
⇨ $(x-y)^2=144$ $=>(x-y)=12$
Hence option (b) is correct.

2. The sum of the series
$1+\frac{1}{2}(1+2)+\frac{1}{3}(1+2+3)+\frac{1}{4}(1+2+3+4)+\cdots$ up to 40 terms, is:
(a) 820 (b) 861
(c) 409 (d) 430

Solution:
$=1+\frac{1}{2}(1+2)+\frac{1}{3}(1+2+3)+\frac{1}{4}(1+2+3+4)+\frac{1}{5}(1+2+3+4+5)+$
$\cdots$ Up to 400 terms
$=1+\left(\frac{1}{2}+1\right)+\left(\frac{1}{3}+\frac{2}{3}+1\right)+\left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}+1\right)+\left(\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\frac{4}{5}+1\right)+\cdots$
$=1+\left(1+\frac{1}{2}\right)+(1+1)+\left(1+1+\frac{1}{2}\right)+(1+1+1)+\left(1+1+1+\frac{1}{2}\right)\ldots$
By adding pairs of terms,
$=2\frac{1}{2}+4\frac{1}{2}+6\frac{1}{2}+8\frac{1}{2}+\cdots+40\frac{1}{2}$
(20 terms)
$=S_n=\frac{20}{2}\left[2\frac{1}{2}+40\frac{1}{2}\right]$
= 430
Hence, option (d) is the correct answer

3. The value of $1^3+2^3+3^3+\cdots+n^3$ is:
(a) $\frac{n(n+1)(2n+1)}{6}$
(b) $\left(\frac{n(n+1)}{2}\right)^3$
(c) $(1+2+3\ldots.+n)^2$
(d) $(1^2+2^2+3^3+\cdots n^2)^2$
Solution:
$1^3+2^3+3^3+\cdots n^3$
Using the method of induction,
Let the number of terms = 3
So, by taking only three terms,
$1^2+2^3+3^3=36$
Put n=3 in all the options.
Using option (a) $\frac{3((3)+1)(2(3)+1)}{6}=28$
Using option (b) $\left(\frac{3(3+1)}{2}\right)^3=216$
Using option (c) $(1+2+3)^2=36$
Since the value of option (c) matches with our value,
Hence option (c) is the correct answer.

4. The sum of 100 terms of $(1-2+3-4+5-\cdots.)$ is:
(a)−50 (b)−500
(c)−100 (d)−1000

Solution:
$1-2+3-4+5-6+\cdots+99-100$
As, it can be seen that the sum of each pair $=-1$
Since for 100 terms, there will be 50 terms present,

⇨ Sum = -1×50
= -50
Hence option (a) is the correct answer.

5. What is the value of x in the sequence 2, 7, 14, 23, 34, ?

(a) 45	(b) 46
(c) 47	(d) 53

Solution:
It can be noticed that the sum of successive terms of the sequence is another sequence of odd numbers starting from 5
i.e. $7-2=5$
$14-7=7$
$23-14=9$
$34-23=11$
So, the next term will be:
$34+13=47$

3D-Geometry

Previous Year Questions

6. $|a^{\rightarrow}+b^{\rightarrow}|^2 - |a^{\rightarrow}-b^{\rightarrow}|^2$ is equal to

(a) 0	(b) $4a^{\rightarrow}b^{\rightarrow}$
(c) $-4\vec{a}\vec{b}$	(d) $a^{\rightarrow}b^{\rightarrow}$

Solution:
$= \left|\vec{a}+\vec{b}\right|^2 - \left|\vec{x}-\vec{b}\right|^2$
$= (\vec{a}+\vec{b})(\vec{a}+\vec{b}) - (\vec{a}-\vec{b})(\vec{a}-\vec{b})$
$= \left((\vec{a}\vec{a}) + (\vec{b}\vec{a}) + (\vec{a}\vec{b}) + (\vec{b}\vec{b})\right) -$
$\left((\vec{a}\vec{a}) - (\vec{b}\vec{a}) - (\vec{a}\vec{b}) + (\vec{b}\vec{b})\right)$
$= 4(\vec{a}\vec{b})$

7. The vectors $\hat{\imath}+2p\hat{\jmath}+4q\hat{k}$ and $\hat{\imath}+4p\hat{\jmath}+2q\hat{k}$ are:
(a) Orthogonal if $p=q$
(b) Orthogonal if $p=-q$
(c) Orthogonal if $p^2=q^2-1$
(d) Never Orthogonal
Solution:
If, $\hat{\imath}+2p\hat{\jmath}+4q\hat{k})$ & $(\hat{\imath}+4p\hat{\jmath}+2\hat{q})$ are perpendicular
Then, $\left(\hat{\imath}+2\hat{p}+4q\hat{k}\right).\left(\hat{\imath}+4p\hat{\jmath}+2q\hat{k}\right)=0$
⇨ $(1+8p^2+8q^2)=0$
$p^2+q^2=-\frac{1}{8}$
∵ Sum f squares can never be $-ve$
∴ the two vectors will never be perpendicular
Hence, option (d) will be correct.

8. If $\vec{a}=i+j+k, \vec{b}=4i+3j+4k$ & $\vec{c}=i+\alpha j+\beta k$ are linearly dependent vectors, and $|\vec{c}|=\sqrt{3}$, then
(a) $\alpha=1, \beta=-1$
(b) $\alpha=1, \beta=\pm 1$
(c) $\alpha=-1, \beta=\pm 1$
(d) $\alpha=\pm 1, \beta=1$

Solution:
$\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$
$\vec{b}=4\hat{\imath}+3\hat{\jmath}+4\hat{k}$
$\vec{c}=\hat{\imath}+\alpha\hat{\jmath}+\beta\hat{k}$
∵ these vectors are linearly independent,
∴ $\sum_{i=1}^{1=3} K_i X_i = 0$
⇨ $\begin{bmatrix}1 & 1 & 1\\ 4 & 3 & 4\\ 1 & \alpha & \beta\end{bmatrix}\begin{bmatrix}C_1\\ C_2\\ C_3\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$
⇨ $\begin{vmatrix}1 & 1 & 1\\ 4 & 3 & 4\\ 1 & \alpha & \beta\end{vmatrix}=0$
This is possible only if the columns or rows are proportional.
⇨ $\alpha=1, \beta=1$
or,
⇨ $\alpha=-1, \beta=1$
So, the answer would be $(\alpha,\beta)=(\pm 1,1)$
Hence, the answer is option (d)